Satisficing the Masses: Applying game theory to large scale, democratic decision problems

Kshanti Greene, Joseph Kniss, George Luger

University of New Mexico

Carl Stern

Management Sciences, Inc.

Outline

1. Goals: What am I trying to accomplish?
2. Challenges: Why is this so difficult?
3. Background
4. Approach
5. Results

Goals

1. Collective Belief Models
2. Social Decision-making

Collective Belief Models

No!!!!

Yes!!!!

Because!!!!!

Collective Belief Models

A tool for capturing and utilizing diverse beliefs

- Generate *collective belief models* that behave rationally within reasonable bounds
- *Define* a collective with respect to belief
- Form an accurate representation of a population using a set of collectives
- Enable rational social decision-making
- Reveal hidden motivators

Social decision-making

Social decision-making

A tool for aiding large-scale decision-making

- Make decisions under uncertainty using the *direct input* of individuals with a stake in the problem
- Incorporate the beliefs and utilities of all involved
- Visualize how decisions affect emerging groups
- Apply game- and decision- theoretic analysis
- Expand survey-taking to include causal foundations
- Evaluate the performance of our leaders

Challenges

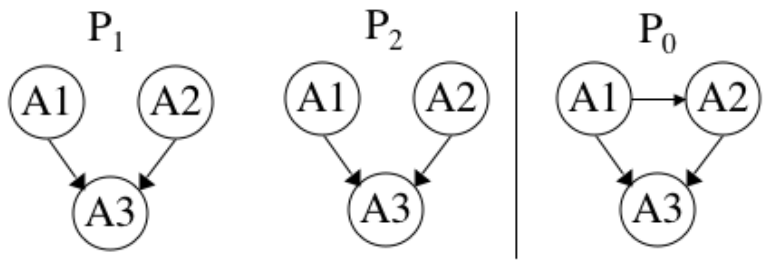
Why is this so difficult?

Divergence

- Divergence in belief causes loss of information in consensus models
  - Average of .8 and .2 versus average of .45 and .55
- Causes increased variance in consensus models
  - Reduces ability to represent beliefs with one distribution
- Consensus models assume cooperation whereas divergence can indicate competitive goals
- Maintaining diversity increases complexity

Impossibility

- Bayesian logic (Pennock & Wellman)
  - Introduces dependencies in the network

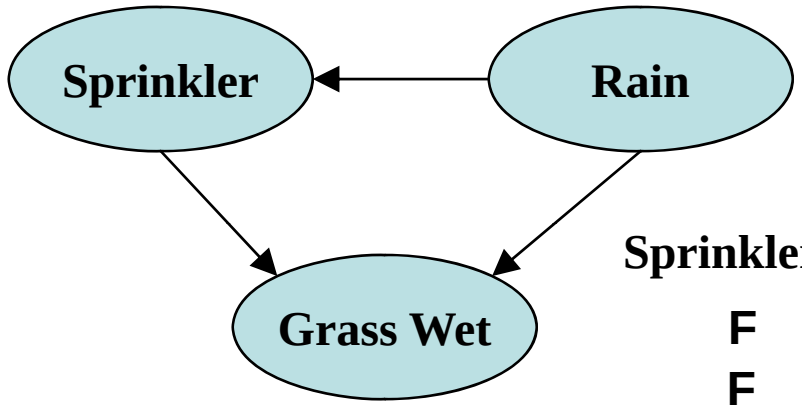


  - Breaks Bayes rule
- Arrow's Impossibility Theorem (Economics)
  - No fair and rational way to aggregate preferences when *3 or more* ranked options
  - Not Pareto optimal!

Background

Bayesian Networks

Rain	Sprinkler T	Sprinkler F
F	0.4	0.6
T	0.01	0.99



Rain T	Rain F
0.2	0.8

Sprinkler	Rain	Grass Wet T	Grass Wet F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

Size = 2^m

Joint probability of network is the product of the *conditional probabilities* at each node:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | Pa_{X_i})$$

$P(W=\text{T}, S=\text{T}, R=\text{F}) = P(R=\text{F})P(S=\text{T} \mid R=\text{F})P(W=\text{T} \mid S=\text{T}, R=\text{F}) =$ 0.29

Pearl, 1988

Bayesian Belief Aggregation

- Combines the beliefs of multiple individuals on graphic model *structure* and *parameters*
- Opinion pool function:

$$P_0 = f(P_1, P_2, \ldots, P_n)$$

Linear OP (LinOP): Weighted arithmetic mean (*average*)

$$P_0(x) = \sum_{i=1}^{N} \alpha_i P_i(x)$$

Logarithmic OP (LogOP): Weighted geometric mean

$$P_0(w_j) = \frac{\prod_{i=1}^{N} [P_i(w_j)]^{\alpha_i}}{\sum_{k=1}^{2^m} \prod_{i=1}^{N} [P_i(w_k)]^{\alpha_i}}$$

Approach

Consensus Belief Clusters for Decision Networks

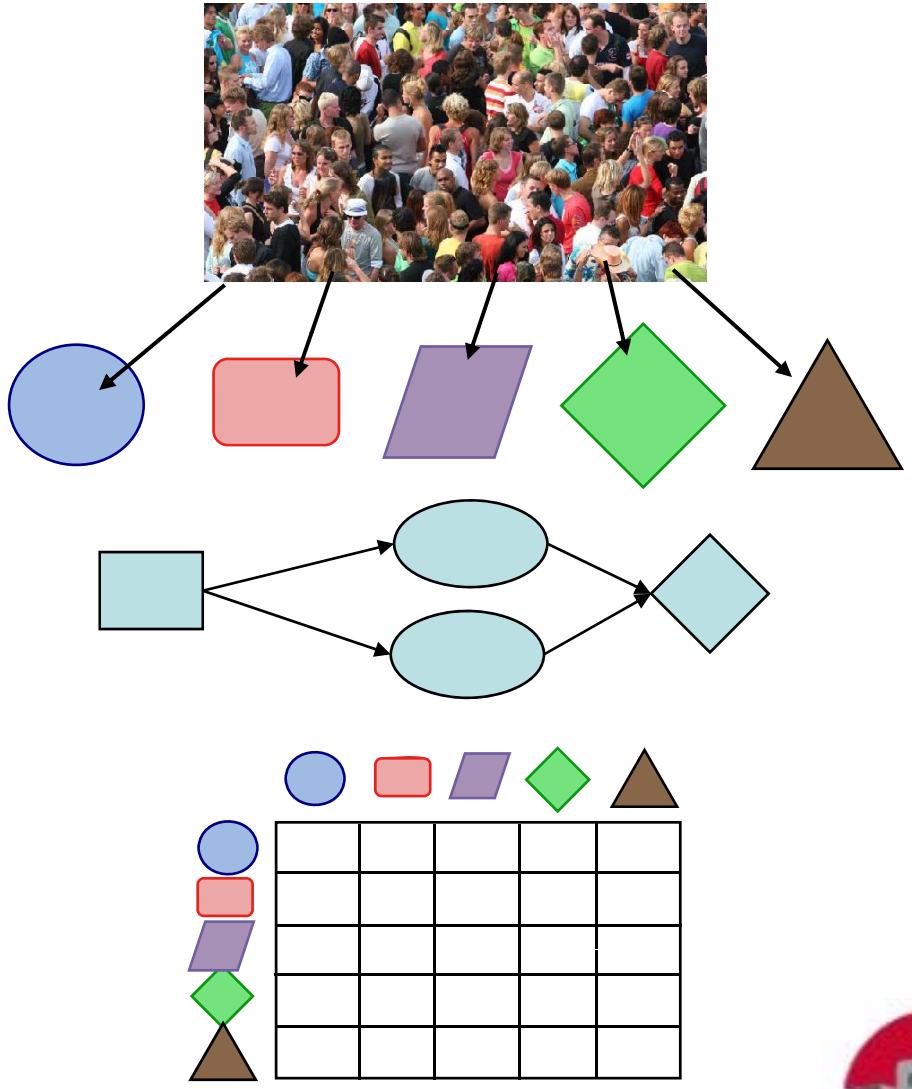
Summary of Approach

Elicit beliefs and utilities

Form collectives

Rank choices using decision networks

Apply game and decision theoretics

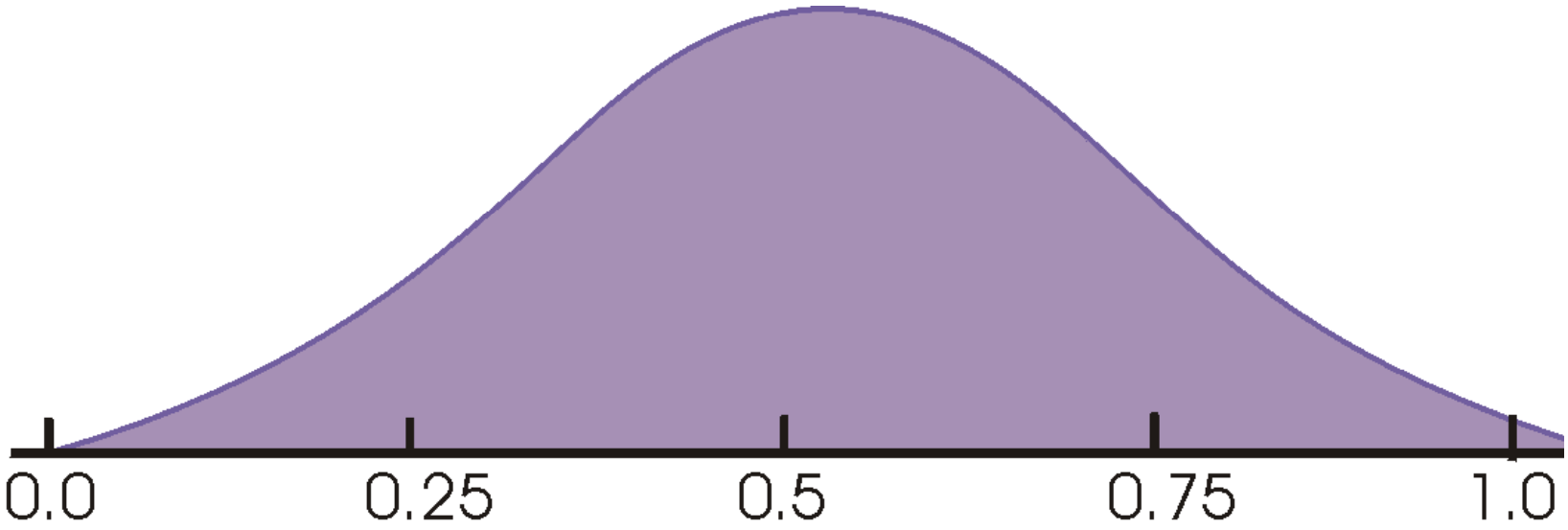


Discovering Collectives

Traditional Opinion Pool

Poll: “What is the likelihood that Obama will win”

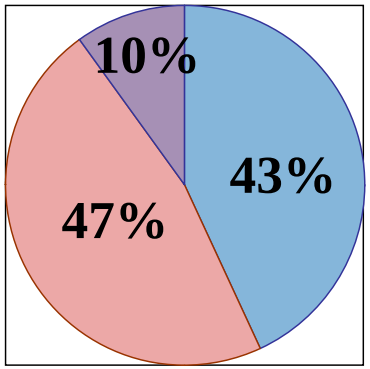
Average among all registered voters: **55%**

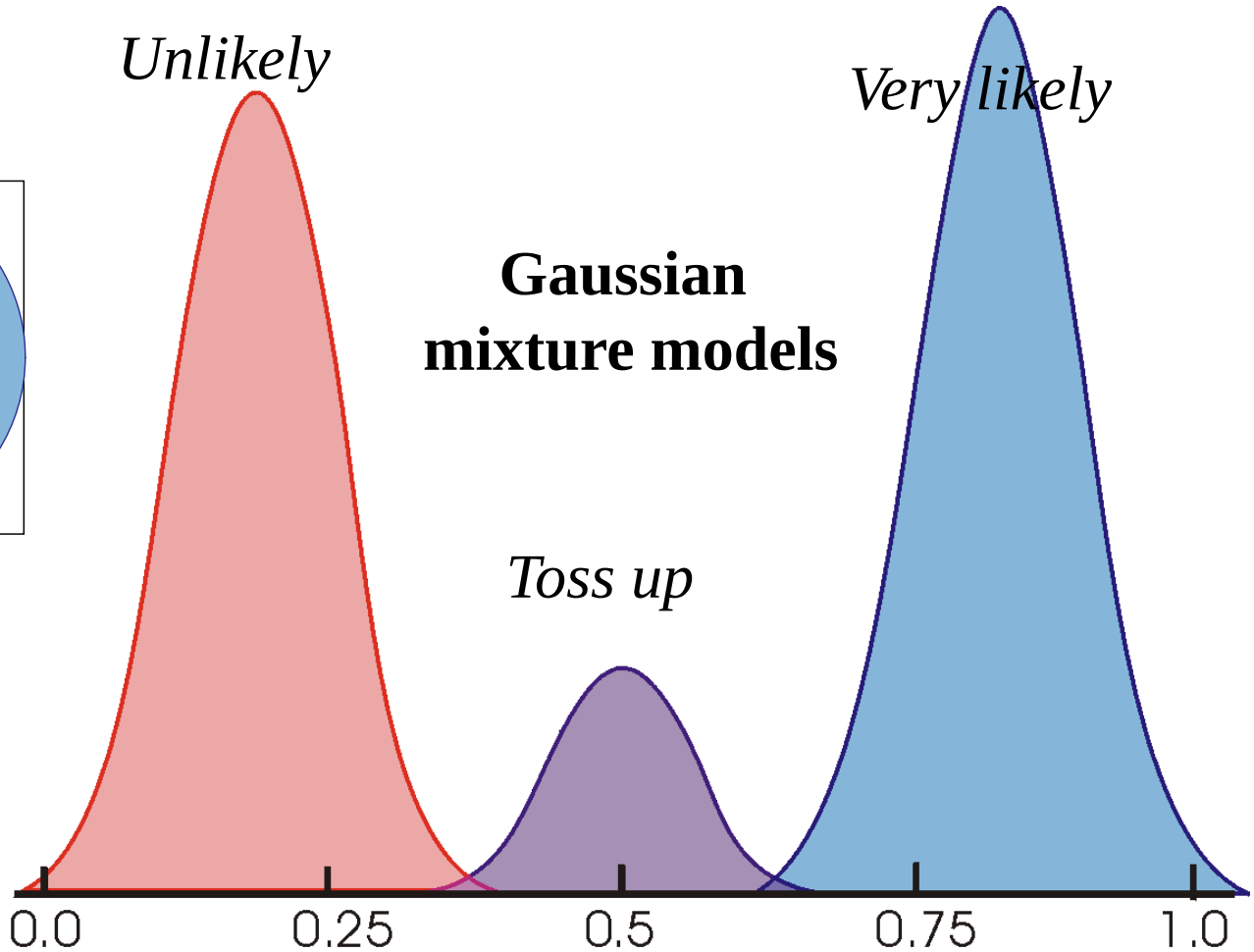


$\mu = 0.55,\ \sigma = very\ large$

Not very informative

Consensus Belief Clusters

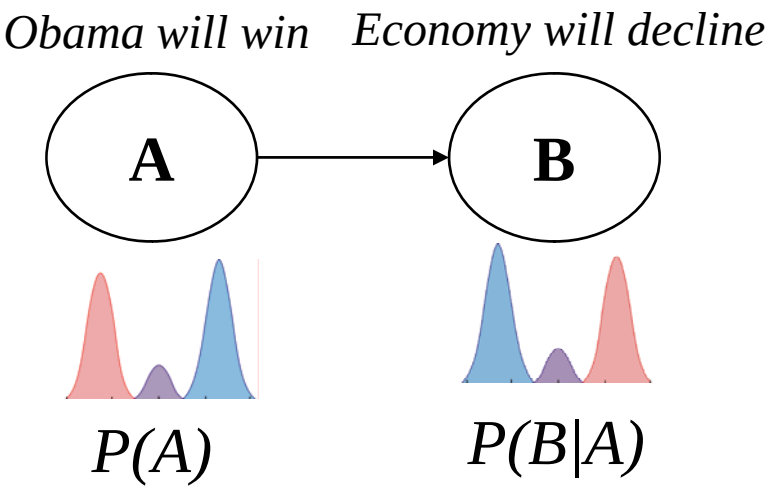




μ more realistically represents groups of consensus

σ restricted to a reasonable variance

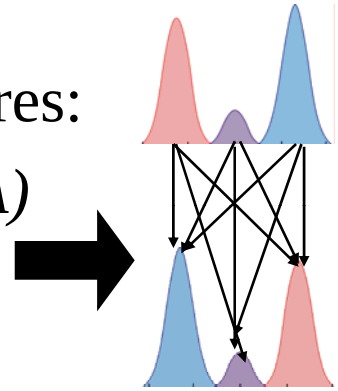
More realism and less variance, but…



Product of two mixtures:

$P(A,B) = P(A)P(B|A)$

9 possible values!



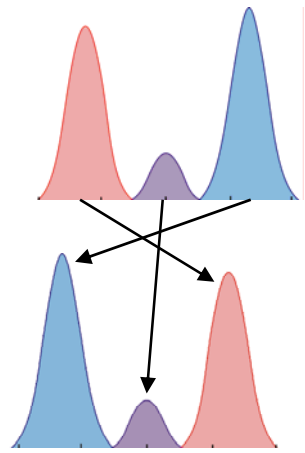
Number of values that need to be propagated is exponential

Leveraging Agreement

- *Observation:* humans that agree on one issue often agree on other related issues
- Can pass values within groups that agree

A = "Obama wins the election"
B = "The economy continues to decline"

$P(A) =$



$P(B|A) =$

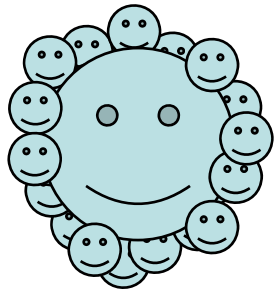
Product of two mixtures:

$$P(A,B) = P(A)P(B|A)$$

Only 3 values!
(Linear!)

What is a collective?

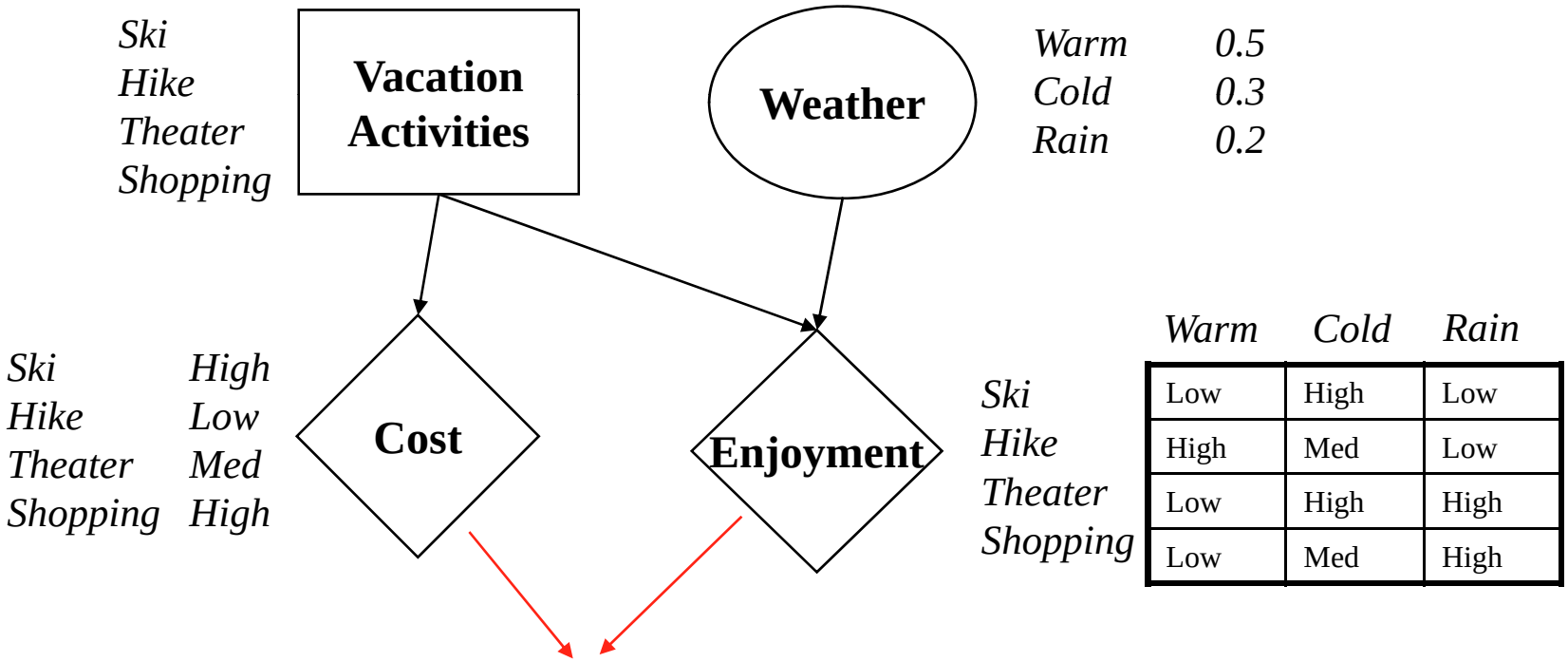
- Goal: Find the groups of individuals that agree *to an extent* such that collective has rational behavior
- What degree of agreement is needed to reduce or eliminate impossibility results?
  - Pennock's theorem:
    - Find an acceptable error bound
  - Arrow's theorem:
    - Satisfiable when "single peakedness" is achieved



- Possible metrics:
  - Standard deviation, entropy, distance function between beliefs, rank order of options

Ranking Choices Using Decision Networks

Form a rank order of options given utilities and uncertainties



Hike, Theater, Ski, Shopping

Applying Decision and Game Theoretics

- Revealing the collectives (G1,G2) enables competitive aspect to emerge
- Decision and game theoretics can then be applied to make appropriate decision:

G1,G2	Hike	Theater	Ski	Shop
Hike	4,2	0,0	0,0	0,0
Theater	0,0	3,4	0,0	0,0
Ski	0,0	0,0	2,3	0,0
Shop	0,0	0,0	0,0	1,1

Results

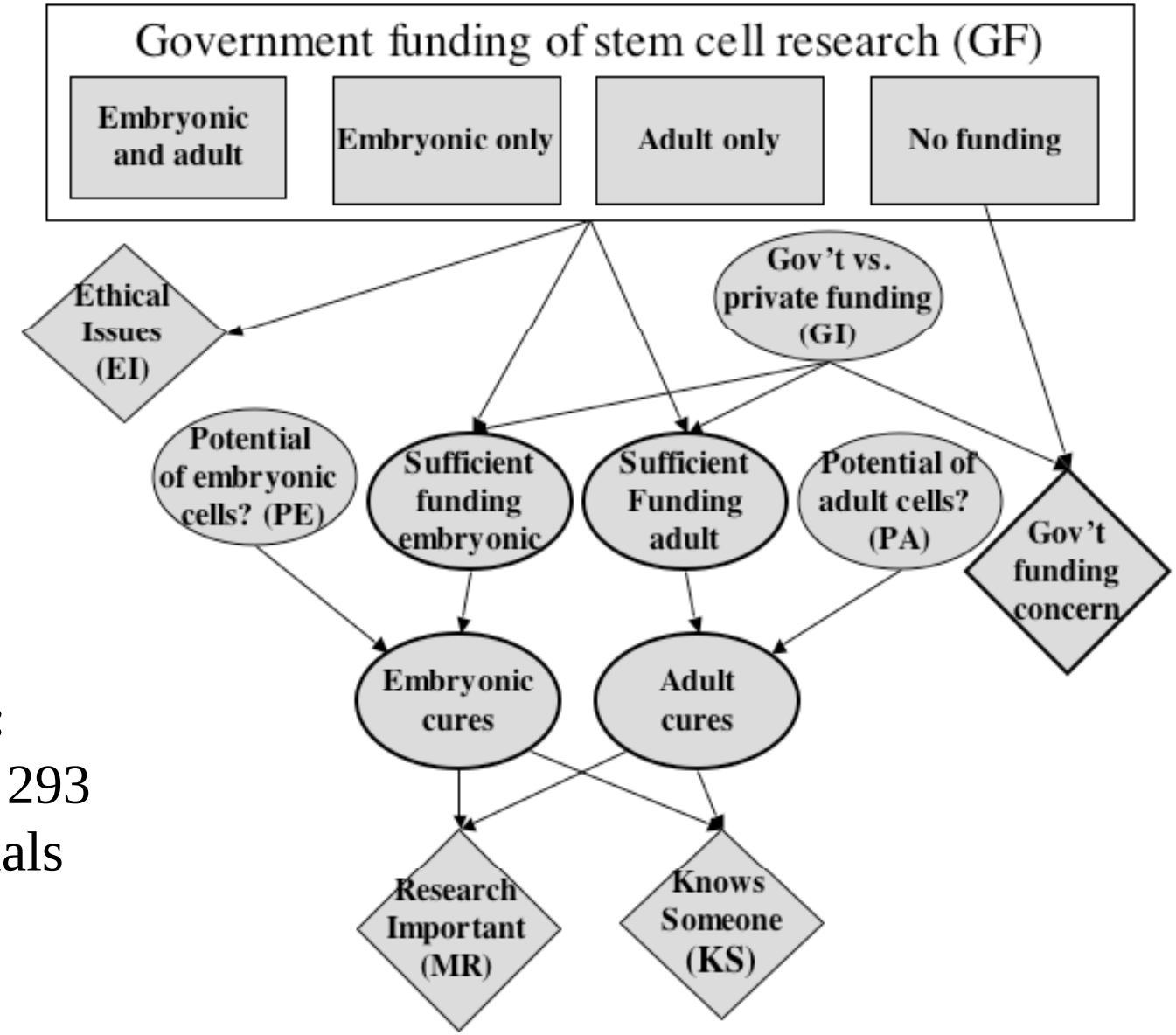
Stem-cell Research Decision Network

Data:

7 beliefs, 293 individuals

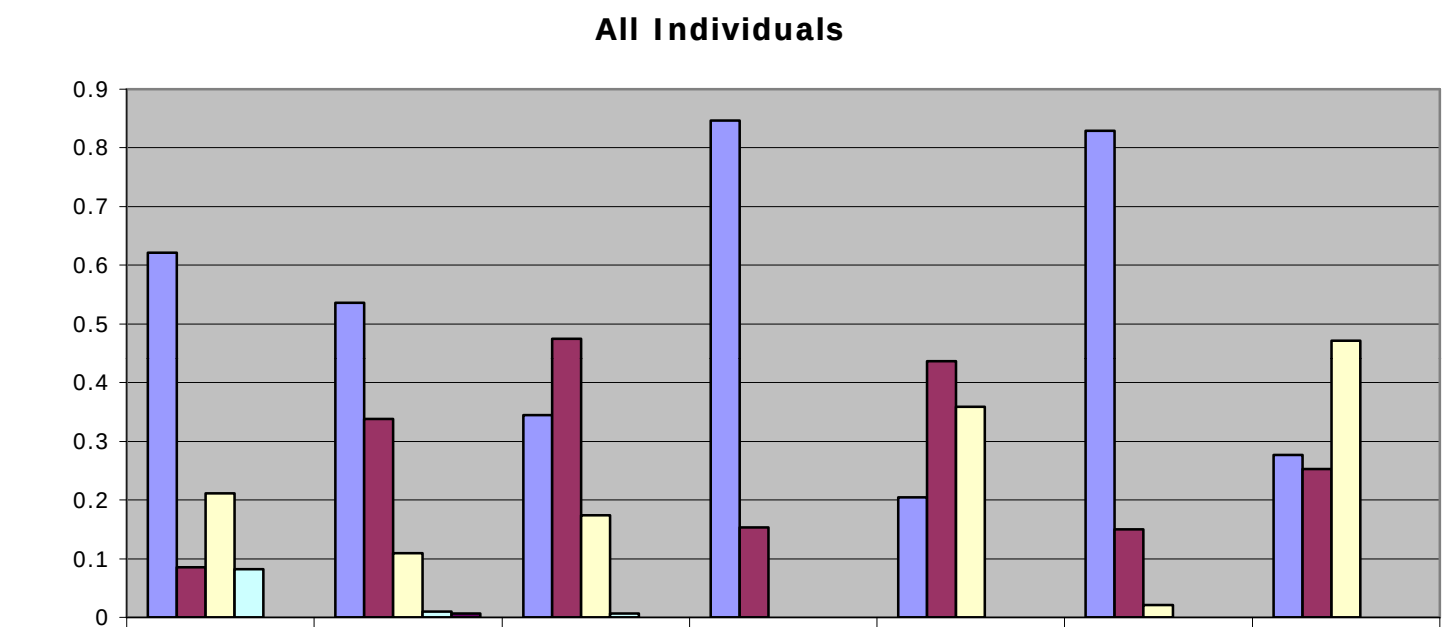
Key	Question	Options
GF	Should the government fund stem cell research?	a. Yes, the government should fund embryonic and adult research b. The government should only fund embryonic stem cell research c. The government should only fund adult stem cell research d. The government should not fund any stem cell research
PE	What is the potential for embryonic stem cell research to cure diseases and provide new methods to test cures?	a. The potential is very high (90%) b. There is some potential (75%) c. It is not clear yet what the potential is (50%) d. It is unlikely to provide cures (25%) e. It is very unlikely to provide cures (10%)
PA	What is the potential for adult stem cell research to cure diseases and provide new methods to test cures?	a. The potential is very high (90%) b. There is some potential (75%) c. It is not clear yet what the potential is (50%) d. It is unlikely to provide cures (25%) e. It is very unlikely to provide cures (10%)
GI	Would government funding of stem cell research improve its potential?	a. Yes, government funding is important to advance research (99%) b. No, private funding is sufficient to advance research (1%)
EI	Are you concerned about the ethical issues of using embryonic stem cells?	a. I am very concerned. The potential does not outweigh the ethical issues. (-20) b. I am a little bit concerned. I think there is potential, but some issues need to be addressed. (-10) c. I am not very concerned. I think the potential outweighs any ethical issues. (0)
MR	How important is advancing medical research and curing diseases in general to you?	a. Very important. (+15) b. Somewhat important. (+10) c. Not very important. (0)
KS	Do you know someone who could potentially benefit from stem cell research?	a. Yes, myself or a loved one (+15) b. Yes, a friend or associate (+10) c. Not personally(0)

Model



Data:
7 beliefs, 293 individuals

Distribution of all beliefs



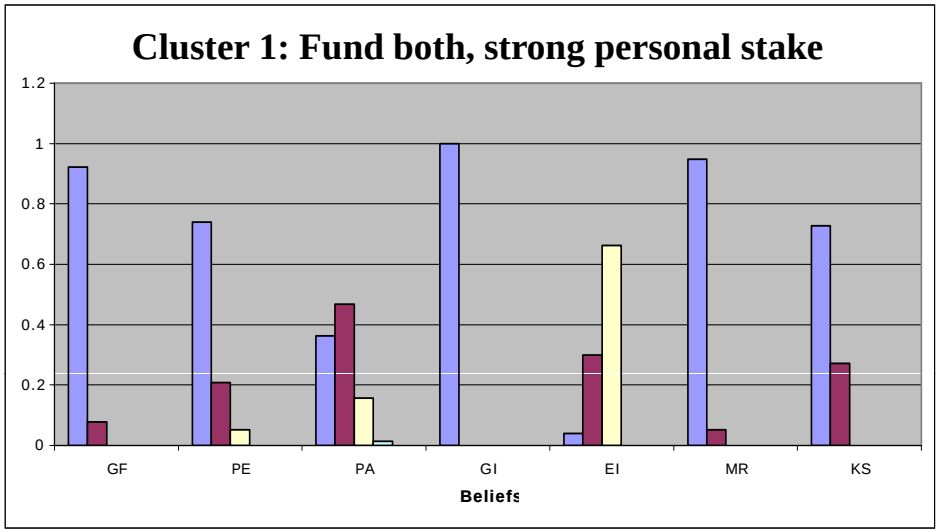
	GF	PE	PA	GI	EI	MR	KS
Series1	0.621	0.5358	0.3446	0.8461	0.2047	0.8291	0.2764
Series2	0.0853	0.3379	0.4743	0.1535	0.4367	0.1501	0.2525
Series3	0.2116	0.1092	0.174	0.0001	0.3583	0.0205	0.4708
Series4	0.0819	0.0102	0.0068	0.0001	0.0001	0.0001	0.0001
Series5	0.0001	0.0068	0.0001	0.0001	0.0001	0.0001	0.0001

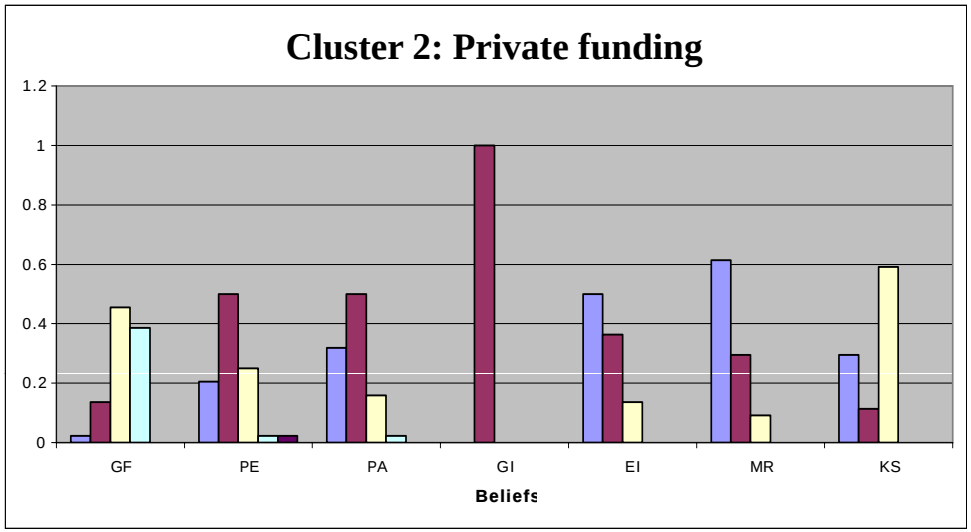
Beliefs

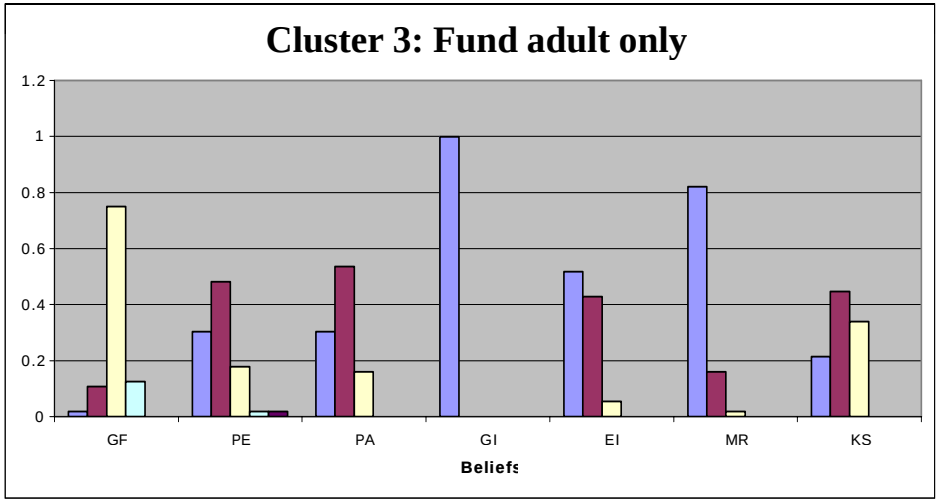
Beliefs diverge to a varying extent

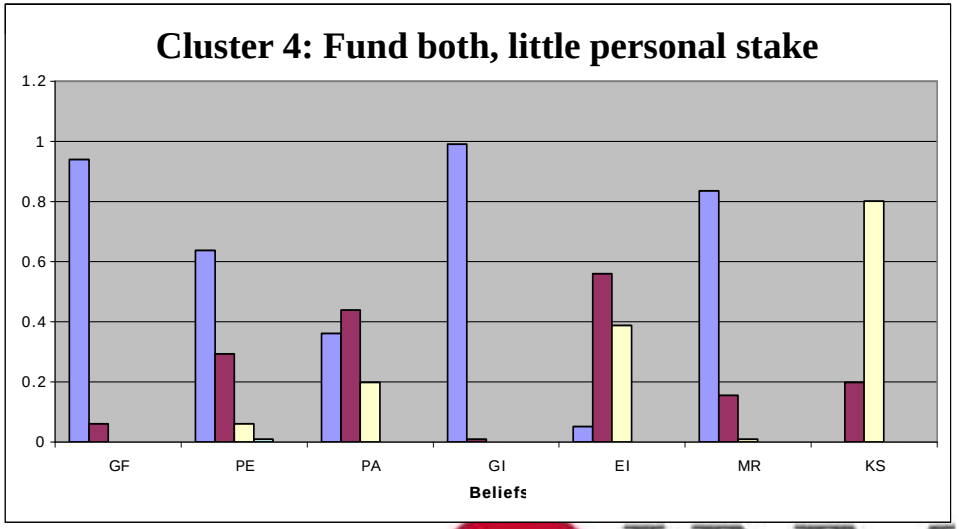
Clustering reveals groups with opposing beliefs

Distributions of clusters









Utilities of consensus vs. groups

Consensus:

Decision option	Utility
Fund embryonic and adult stem cell research	*16.3*
Fund embryonic stem cell research only	6.1
Fund adult stem cell research only	13.8
Do not fund any research	3.8

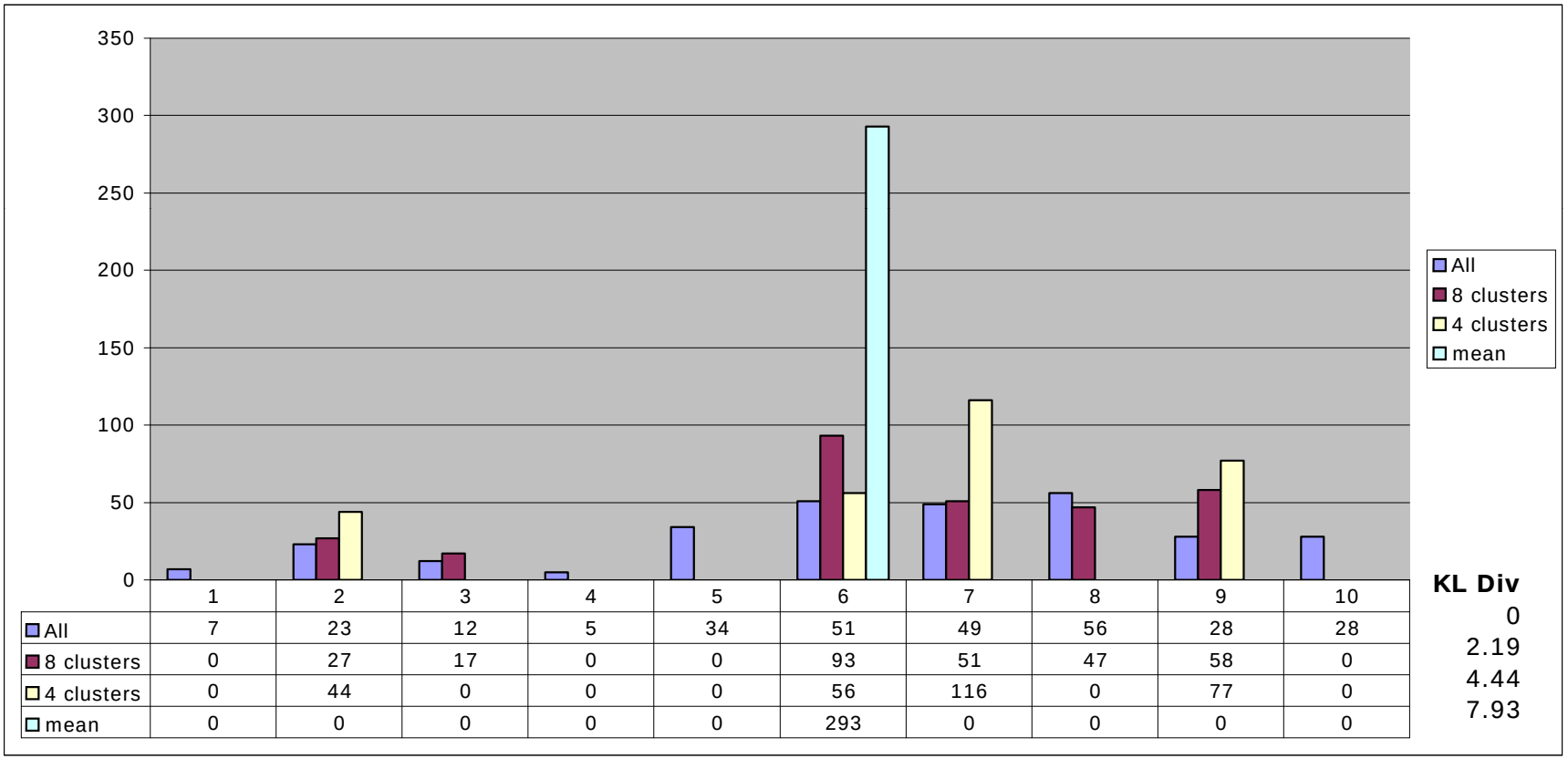
Clusters:

Decision	Cluster				Pareto Optimality
	Cluster 1	Cluster 2	Cluster 3	Cluster 4	
Fund Both	38.64	-19.42	14.81	16.74	Optimal
Fund Embryo only	18.82	-9.29	-0.15	5.75	Not optimal
Satisficing? *Fund adult only*	19.83	3.35	14.97	11.17	Optimal
Fund neither	0.01	19.45	0.01	0.17	Optimal
Characteristics					
Size	77	44	56	116	
Percent	26%	15%	19%	40%	
Optimal solution	Fund both	Fund neither	Adult only	Fund both	
Worst solution	Fund neither	Fund both	Embryo only	Fund neither	

Accurate representation?

All, 8 clusters, 4 clusters, Mean

Decision Option: *Fund Both*



	1	2	3	4	5	6	7	8	9	10	KL Div
All	7	23	12	5	34	51	49	56	28	28	0
8 clusters	0	27	17	0	0	93	51	47	58	0	2.19
4 clusters	0	44	0	0	0	56	116	0	77	0	4.44
mean	0	0	0	0	0	293	0	0	0	0	7.93

Divergence from *All* decreases with more clusters

Next Steps

- How much agreement is needed to form a rational collective?
  - Same rank ordering?
- How do we find the appropriate collectives a priori
- What are the bounds to achieve rationality using Arrow's theorem?
  - Hypothesis: Since a decision with 2 options has a rational result, what if individuals in collective agree within one rank order?

A= 1 2 3	A= 1 2 3
B= 3 2 1	B= 1 3 2
C= 1 3 2	C= 1 2 3
Not OK	*OK?*

- Crossover between game theory and Arrow's theorem?

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Additional slides

Related Work

- Topological Fusion: (Matzkevich and Abramson, 1992)
  - Merge partial networks
  - Prior vs. posterior compromise
- Pennock and Wellman (‘96, ‘97, ‘99, ‘05):
  - Proved impossibility of opinion pool functions
  - Market-based approaches: individuals buy stocks to support beliefs
    - *Move towards a consensus over time*
    - *Highly sensitive to risk averseness*
- Combining Rules (Pearl, etc..)
  - Enable forming models from independent parent-child relationships
- Collaborative Filtering
  - No causal information or dependencies
- Voting
  - Kenneth Arrow’s impossibility theorem

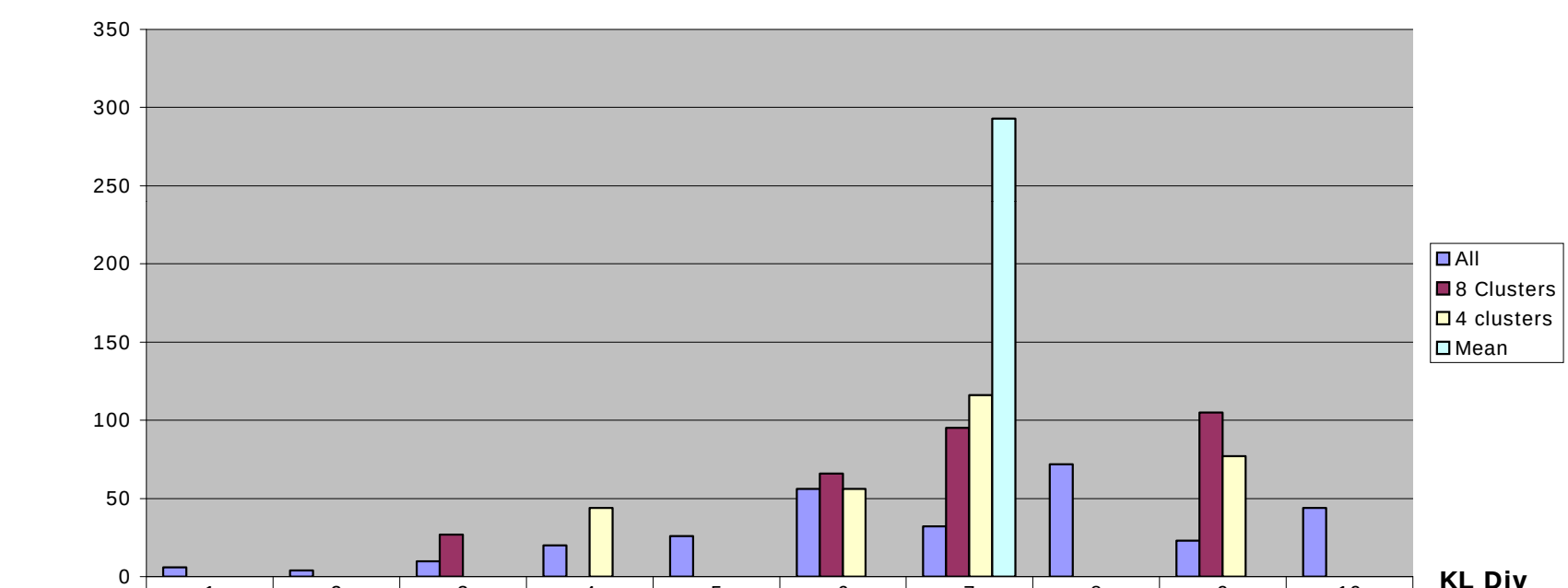
Are we achieving an accurate representation?

Metric: *KL Divergence* compares overlap of two distributions

Compare distributions:

All, 8 clusters, 4 clusters, Mean

Decision Option: *Fund Embryonic only*



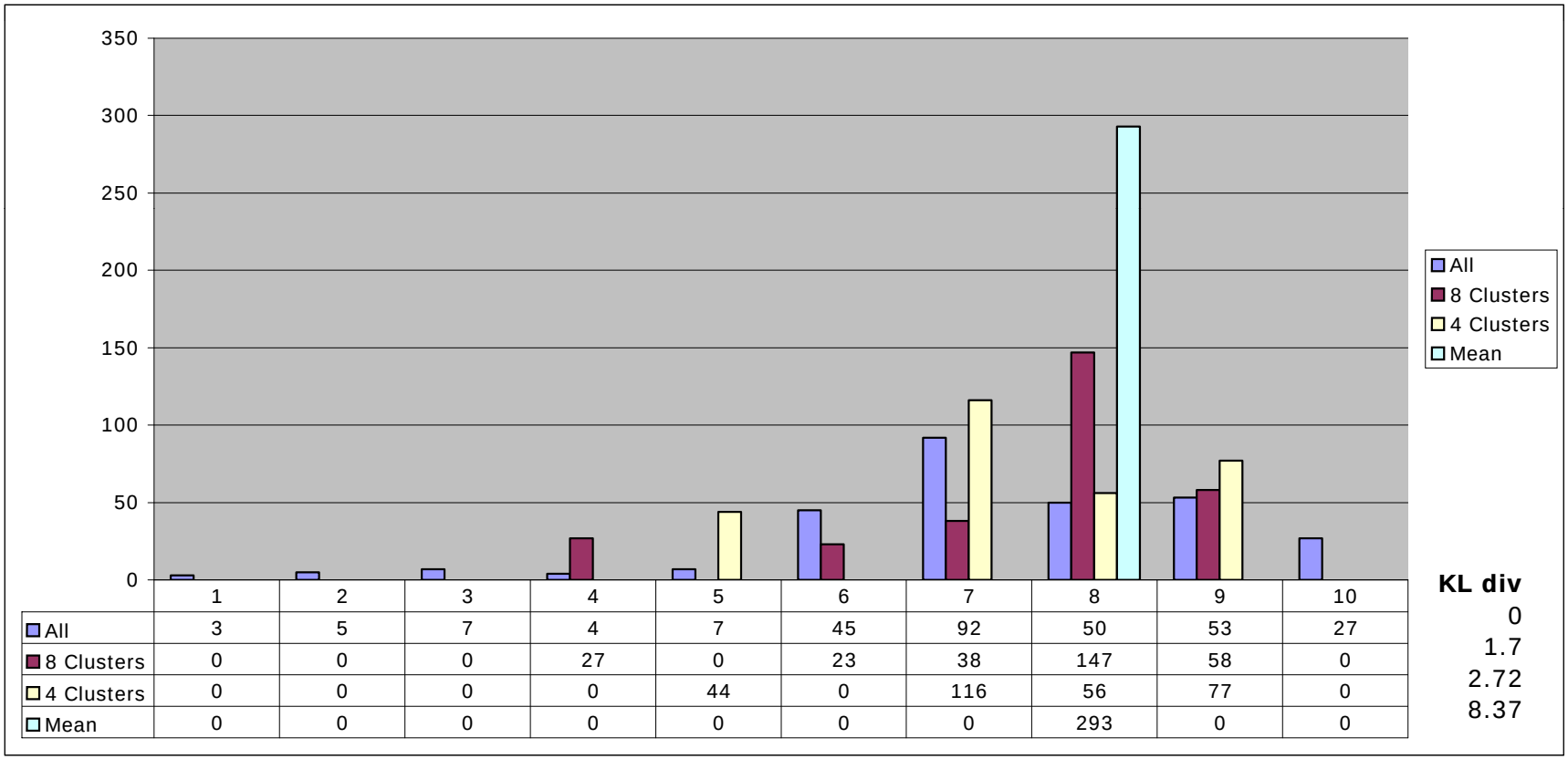
	1	2	3	4	5	6	7	8	9	10	KL Div
All	6	4	10	20	26	56	32	72	23	44	0
8 Clusters	0	0	27	0	0	66	95	0	105	0	5.68
4 clusters	0	0	0	44	0	56	116	0	77	0	5.35
Mean	0	0	0	0	0	0	293	0	0	0	8.9

In this case 4 clusters did better than 8

Compare distributions:

All, 8 clusters, 4 clusters, Mean

Decision Option: *Fund Adult only*



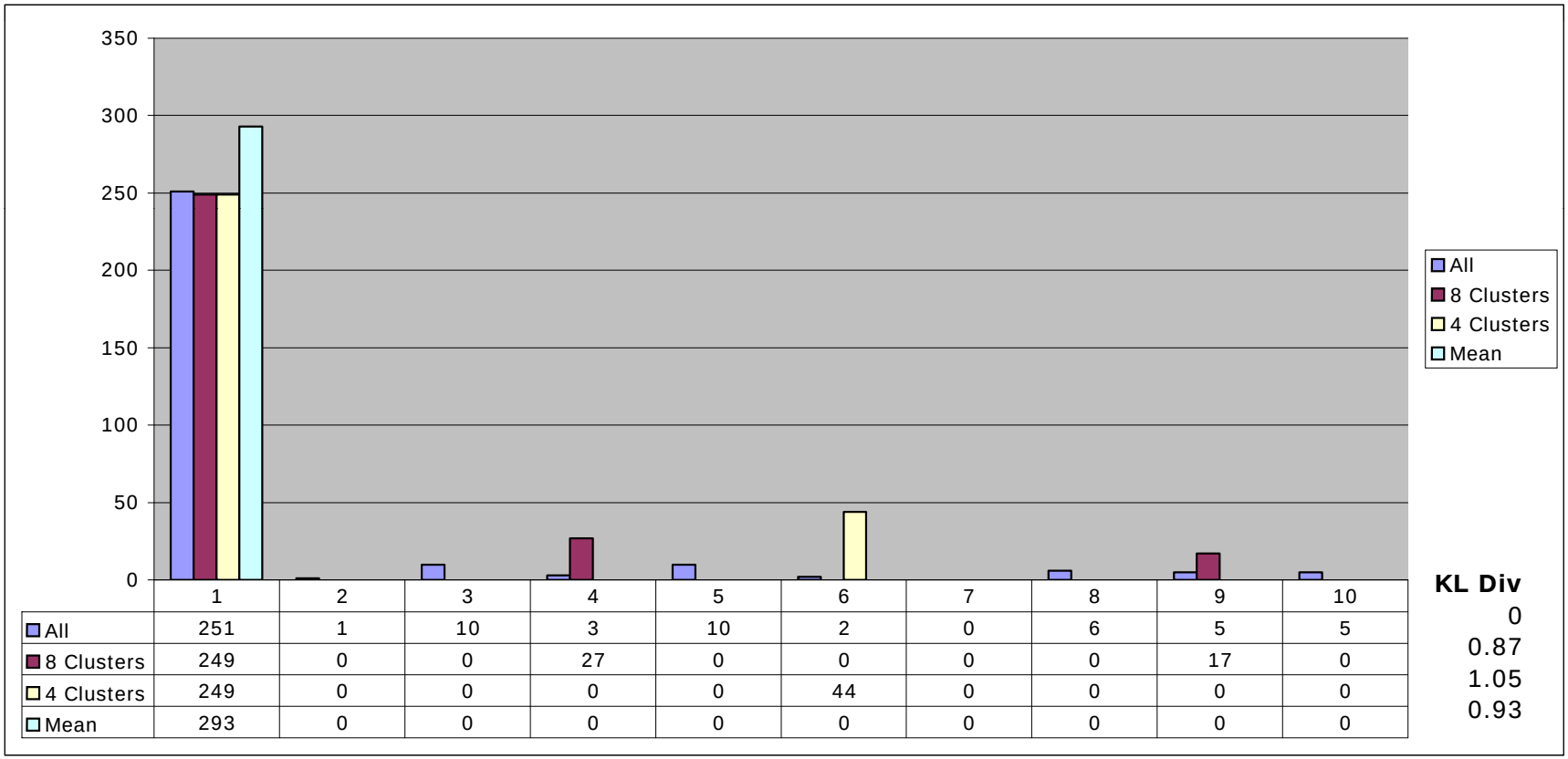
	1	2	3	4	5	6	7	8	9	10	KL div
All	3	5	7	4	7	45	92	50	53	27	0
8 Clusters	0	0	0	27	0	23	38	147	58	0	1.7
4 Clusters	0	0	0	0	44	0	116	56	77	0	2.72
Mean	0	0	0	0	0	0	0	293	0	0	8.37

Divergence from *All* decreases with more clusters

Compare distributions:

All, 8 clusters, 4 clusters, Mean

Decision Option: *Fund neither*



	1	2	3	4	5	6	7	8	9	10	KL Div
All	251	1	10	3	10	2	0	6	5	5	0
8 Clusters	249	0	0	27	0	0	0	0	17	0	0.87
4 Clusters	249	0	0	0	0	44	0	0	0	0	1.05
Mean	293	0	0	0	0	0	0	0	0	0	0.93

In this case one mean did better than 4 clusters

Summary

- Forcing cooperation through consensus models is not rational and may be better represented by a competitive model
- Competitive models enable us to determine a pareto optimal solution
- In strategic games we can also find minimax solutions and Nash equilibrium
- Reveals satisficing solutions not found using consensus models